The Duality Relationship between Minimizing the Probability of Ruin and Controller Stopper Games

We reveal an interesting convex duality relationship between two problems: (a) minimizing the probability of lifetime ruin when the rate of consumption is stochastic and when the individual can invest in a Black-Scholes financial market; (b) a controller-and-stopper problem, in which the controller controls the drift and volatility of a process in order to maximize a running reward based on that process, and the stopper chooses the time to stop the running reward and rewards the controller a final amount at that time. Our primary goal is to show that the minimal probability of ruin, whose stochastic representation does not have a classical form as does the utility maximization problem (i.e., the objective's dependence on the initial values of the state variables is implicit), is the unique classical solution of its Hamilton-Jacobi-Bellman (HJB) equation, which is a non-linear boundary-value problem. We establish our goal by exploiting the convex duality relationship between (a) and (b).

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